NOVA SCHOOL OF BUSINESS & ECONOMICS

Mathematics Written Exam

Date	Duration	Elements of consultation allowed
28 May 2021	2 hours	Calculator

Instructions

The exam is composed by two compulsory groups.

Group I is composed by multiple choice questions and Group II is composed by open answer questions.

For each multiple choice question, four answer options are presented, of which exactly one is correct. Indicate your answer by selecting the letter corresponding to the option that you consider to be correct. Each correct, incorrect or null answer in Group I has the classification of, respectively, 1/20, −0.33/20 and 0/20 points. In case more than one option is selected for the same question, the answer will be considered null. A negative total for this set of questions adds zero to the final grade.

All open answer questions should be carefully justified, and all computations should be presented. The classification for each question in this group is indicated next to it.

Do not unstaple this book.

Do not use any type of corrector. If necessary, cross out.

Name of the Candidate

Group I

1 Let Ω be the sample space of a random experiment, and let A and B be two events in Ω. Suppose that $P(A) = 0.3$, $P(A \cup B) = 0.6$ and $P(A \mid B) = 0.3$. What is the value of $P(A \cap B)$?

a) 0.61

b) 0.43

c) 0.13

d) None of the above.

2 500 debit cards are being kept in a safe deposit and are ready for distribution. Exactly 50 of them are defective. A sample of 10 cards was randomly selected, with replacement. What is the probability that no card in that sample is defective (consider the rounding to four decimal places)?

a) 0.9624

b) 0.3487

c) 0.1293

d) None of the above.

3 In a certain industrial sector, a study of significant finantial difficulties faced by some companies is being conducted. Companies were classified as small (50%), medium-sized (35%) and large; 53.5% of all companies have finantial difficulties, 60% of all small companies have finantial difficulties, and 17.5% of all companies are medium-sized and have finantial difficulties. What is the probability that a company with finantial difficulties is large (consider the rounding to two decimal places)?

a) 0.06

b) 0.08

c) 0.11

d) None of the above.

4 What is the domain of the function f defined by $f(x) = \frac{\pi - 2\sqrt{e^{-x}(9-x^2)}}{(x^2-x-6)\sqrt[5]{1-x}\ln(|e^x-1|)}$?

a) $]-\infty, 1[\setminus \{-2,0\}$

b) $[-3,1[\setminus \{-2\}$

c) $[-3,3[\setminus \{-2,0,1,\ln(2)\}$

d) None of the above.

5 What is the limit, in case it exists, of the sequence (u_n) defined by

$$u_n = \begin{cases} \frac{1}{\ln\left(\frac{1}{n}\right)} + \left(\frac{n-2}{n+3}\right)^{n-1} \times n \sin\left(\frac{1}{n}\right) + e^{-n}, & \text{se } n \geq 5 \\ \frac{4^n - 2}{n + 3 \times 2^{n+3}}, & \text{se } n < 5 \end{cases} ?$$

a) 1

b) e^{-1}

c) The limit does not exist.

d) None of the above.

6 Consider, in an orthonormal referential $Oxyz$, the plane α defined by $6x - y + 8z = 2$. Let r be the perpendicular line to the plane α that goes through the point P of coordinates $(3,0,4)$. Which of the following systems of equations defines the line r?

a) $\frac{x-6}{3} = \frac{z-8}{4} \wedge y = 0$

b) $\frac{1}{3}x = 1 - 2y = \frac{1}{4}z$

c) $\frac{x-3}{6} = \frac{z-4}{8} \wedge y = 0$

d) None of the above.

7 Which of the following conditions defines, on the complex plane, a perpendicular line to the half-line defined by $\arg(z) = \frac{7\pi}{4}$?

a) $\operatorname{Re}(z) = \frac{\sqrt{2}}{2}$

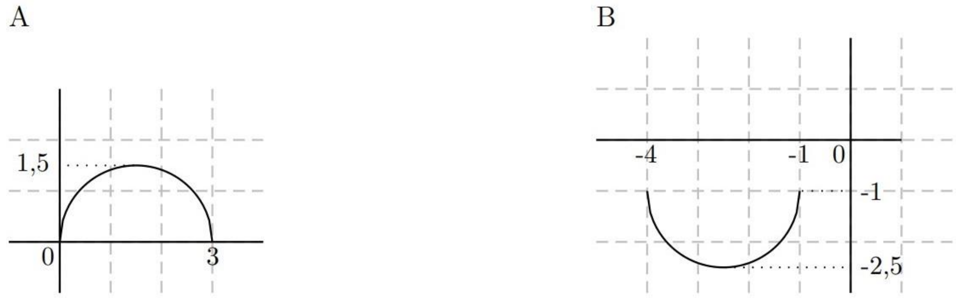
b) $|z| = |z - 1 - i|$

c) $|z - 1 - i| = |z + 1 - i|$

d) None of the above.

Group II

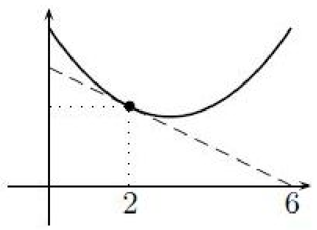
1 (1/20) In figure A, it is represented the graph of the function $f(x) = \sqrt{3x - x^2}$. In figure B, it is represented a curve obtained by the application of geometrical transformations to the graph in figure A. Caracterize the real function of real variable whose graph is the one represented in figure B, justifying your answer conveniently.



2 (1.5/20) Using exclusively analytical methods, solve, in $\mathbb{R}$, the inequation

$$-e^{-2x^3}\left(\frac{1}{9}\right)^{2-x^2} + e^{-4x}\left(\frac{1}{3e^x}\right)^{2x^2-4} < 0.$$

3 (1.5/20) The figure below represents the graph of a function f and the tangent line to the graph of f at the point (2,2).



Consider $g(x) = \frac{1}{f(x)} - x(f(x))^4$. Find the value of $g'(2)$, justifying your answer.

4 (1/20) Consider the sequence (u_n) defined by recurrence by

$$\begin{cases} u_1 = 2 \\ u_{n+1} = (-1)^n u_n + \dfrac{u_n}{2n-1} \end{cases}.$$

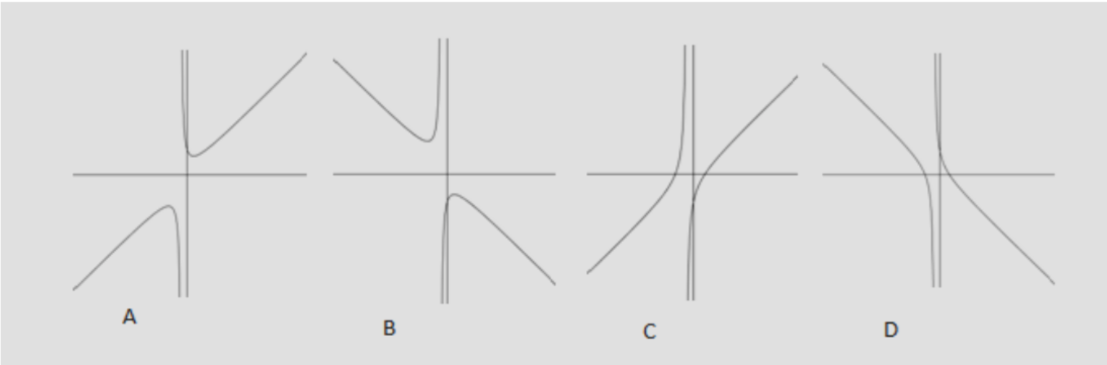
Suppose that (u_n) is a convergent sequence and find its limit. Justify your answer conveniently.

5 (1.2/20) Using exclusively analytical methods, solve the equation

$$\tan(\mathrm{x}) + \frac{1}{\cos(x)} = 2\cos(x),$$

on the real variable x, presenting all your computations.

6 (1/20) Consider the function g defined by $g(x) = \frac{x^2+x+4}{x+1}$. Using anaylical methods, find which of the following graphs may represent the function g. Present all your computations.



7 Consider the function $f: \mathbb{R} \longrightarrow \mathbb{R}$ defined by

$$f(x) = \begin{cases} x\ln(x), & \text{se } x > 0 \\ e^{\sqrt{-x}} + k, & \text{se } x \leq 0 \end{cases}$$

with k a real constant. Using exclusively analytical methods:

a) (0.8/20) Find k such that the function f is continuous in all its domain. Justify.

b) (1/20) Show that there exists a unique point c in $]1,2[$ such that $f(c) = \ln(3)$.

c) (1/20) Consider $k = 2$. Write the equation of the tangent line to the graph of f at the point $x = -1$.

8 (1.5/20) Let $f: \mathbb{R}^+ \longrightarrow \mathbb{R}$ be a function with derivative given by

$$f'(x) = (1-x)^3 \ln(e^x - 1).$$

Using exclusively analytical methods, study f regarding monotonicity and the existence of local extrema, classifying them as minima or maxima points. Justify conveniently.

9 (1.5/20) Using exclusively analytical methods, find the dimensions of the right triangle of maximal area, among those with hypotenuse equal to $2\sqrt{2}$ meters. Present all your computations.

Formulae

Special Limits

$$\lim\left(1+\frac{1}{n}\right)^n = \mathrm{e} \qquad (n \in \mathbb{N})$$

$$\lim_{x\to 0}\frac{\operatorname{sen}(x)}{x} = 1$$

$$\lim_{x\to 0}\frac{e^x-1}{x} = 1$$

$$\lim_{x\to 0}\frac{\ln(x+1)}{x} = 1$$

$$\lim_{x\to +\infty}\frac{\ln(x)}{x} = 0$$

$$\lim_{x\to +\infty}\frac{e^x}{x^p} = +\infty \qquad (p \in \mathbb{R})$$

Derivation Rules

$$(u+v)' = u'+v'$$

$$(uv)' = u'v+uv'$$

$$\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$$

$$(u^n)' = nu^{n-1}u' \qquad (n \in \mathbb{R})$$

$$(a^u)' = \ln(a)\,a^u u' \qquad (a \in \mathbb{R}^+ \setminus \{1\})$$

$$(\log_a(u))' = \frac{u'}{\ln(a)\,u} \qquad (a \in \mathbb{R}^+ \setminus \{1\})$$

$$(\operatorname{sen}(a))' = a'\cos(a)$$

$$(\cos(a))' = -a'\operatorname{sen}(a)$$